Letters to the Editor

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Velocity of sound in weak plasmas

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A simple mathematical analysis has been derived basing on the kinetic equations of a gas for evaluating the velocity of an acoustic wave through a weak plasma. The analysis shows a good agreement with the previously established results by other workers, for only at low degree of ionization in the gas. It has been found that the velocity of sound increases with the increase in the degree of ionization or temperature of the gas.

When an acoustic wave propagates through an ionized gas there will be a change in its velocity from that of a neutral gas. This change occurs owing to the collisions between the charged particles and the neutral atoms in the gas. The dominant process is the electron-neutral atom, elastic collisions. Ingard *et al* (1966, 1967, 1969) have made theoretical calculations and have shown that the sound waves get amplified due to perturbations in the gas density which leads to a change in the plasma electron density and Kaw (1969) has shown that there will be an increase in the equilibrium temperature due to collisions.

The same conclusions can be drawn for a weak plasma or a slightly ionized gas applying the general kinetic equations of a gas. In a plasma, the total pressure is given by,

$$p = n_0kT_0+n_ekT_e+n_ikT_i \, , \qquad \ldots \quad (1)$$

where n_0, n_i and n_e, T_0, T_i and T_e respectively represent the neutral atom, ion and electron particle density and temperature. k is the Boltzmann constant. For a plasma when $n_i = n_e$ and $T_i = T_0$, the pressure is reduced to

$$p = \frac{\rho}{m_0} kT_e(r+x) \qquad \ldots \quad (2)$$

where $r = (T_0/T_e)$ and $x = n_e/(n_e+n_0)$, x is the degree of ionization in the plasma. The mass density $\rho = (n_e+n_0)m_0$, m_0 being the atomic mass of the gas. An

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acoustic wave passing through a plasma introduces a pressure perturbation because of which there will be a variation in the temperature. Hence,

$$\frac{dT_e}{T_e} = \frac{dp}{p} - \frac{d\rho}{\rho} - \frac{(dr+dx)}{r+x}, \qquad \ldots (3)$$

$$dx = x\left[\frac{dn_e}{n_e} - \frac{d\rho}{\rho}\right],$$

and

$$dr = r\left[\frac{dT_0}{T_0} - \frac{dT_e}{T_e}\right].$$

Substitution of these values of dx and dr into the eq. (3) gives,

$$\frac{dp}{p} = \left(\frac{x}{r+x}\right)\left(\frac{dT_e}{T_e} + \frac{dn_e}{n_e} + \left(\frac{r}{r+x}\right)\left(\frac{dT_0}{T_0} + \frac{d\rho}{\rho}\right)\right. \qquad \ldots (4)$$

The electron-neutral atom elastic collision frequency ν is proportional to the neutral particle density n_0 and the collision frequency at T_e is given by $\nu = \nu_0(T_e/T_0)^{\frac{1}{2}}$; ν_0 being the frequency at $T_e = T_0$.

$$\frac{d\nu}{\nu} = \frac{1}{2}\left(\frac{dT_e}{T_e} - \frac{dT_0}{T_0}\right), \qquad \ldots (5)$$

and

$$\frac{dT_e}{T_e} = \left(2\frac{dn_0}{n_0} + \frac{dT_0}{T_0}\right). \qquad \ldots (6)$$

In an adabatic approximation,

$$\frac{\Delta n_0}{n_0} = \left(\frac{I}{\gamma}\right)\frac{\Delta p}{p} \text{ and } \frac{\Delta T_0}{T_0} = \left(\frac{\gamma-1}{\gamma}\right)\frac{\Delta p}{p} \qquad \ldots (7)$$

where γ is the ratio of the specific heats of the neutral gas.

If the slightly ionized gas is in thermodynamic equilibrium, the electron density of the plasma can be obtained from Saha's equation (Sodha & Palumbo 1966) and

$$\frac{dn_e}{n_e} = \frac{I}{\gamma}\left[\tfrac{1}{2} + \left(\tfrac{3}{4} + \frac{B}{T_0}\right)(\gamma-1)\frac{dp}{p}\right] \qquad \ldots (8)$$

where $B = (eV_i/600\,k)$, Vi is the ionization potential of the gas in volts.

Substituting the values of (dT_e/T_e), (dn_e/n_e) and (dn_0/n_0) eq. (4) can be written as,

$$\frac{dp}{d\rho} = \frac{p}{\rho}\left[\frac{r}{\left(\frac{2r-3x}{2r}\right) - \left(\tfrac{3}{4} + \frac{B}{T_0}\right)\left(\frac{\gamma-1}{\gamma}\right)x}\right]. \qquad \ldots (9)$$

Hence velocity of the acoustic wave through the plasma is given by,

$$V_p = \left(\frac{dp}{d\rho}\right)^{\frac{1}{2}} = \left[\frac{kT_e(r+x)}{m_0}\right]^{\frac{1}{2}} \left[\frac{r\gamma}{(r-(3/2)x-\left(\frac{3}{2}+\frac{B}{T_0}\right)(\gamma-1)x}\right] \quad \ldots \quad (10)$$

It can be seen that for $r = 1$ and $x = 0$, eq. (10) reduces to the general equation for sound velocity in a gas. The degree of ionization can be calculated from von Engel (1965),

$$p\left(\frac{x^2}{1-x^2}\right) = 4{\cdot}9\,\frac{g_{ion}}{g_{gas}}\,10^{-4}.T^{5/2}.\exp(-eV_i/kT). \quad \ldots \quad (11)$$

The gas pressure p is given in torr and the temperature in °K. g's are the statistical weights. x has been calculated for $T \simeq T_e$ from eq. (11) and is shown in figures 1 and 2.

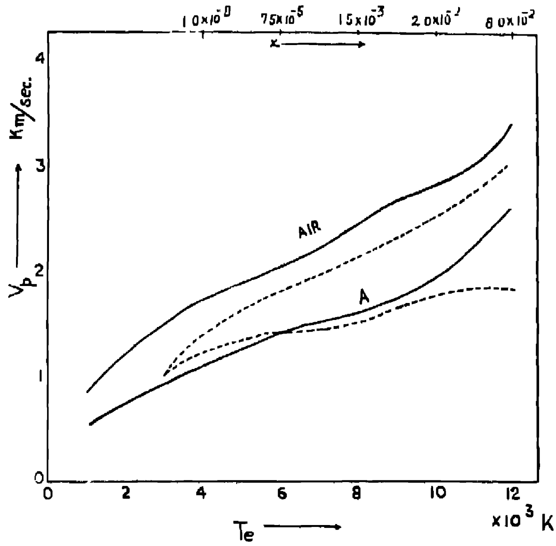

Fig. 1. Variation of sound velocity V_p with the temperature of the plasma for air and argon at 1 atm. and $r = 1$. Dotted curves are derived from Ahloborn's theory. Degree of ionization corresponding to the temperature of the plasma (argon) is shown on the upper scale.

Theoretical curves for air and argon at a pressure of one atmosphere have been shown in figure 1. r is taken to be equal to one. The variation in the sound velocity with the gas temperature (or degree of ionization) shows that the velocity of sound increases with the degree of ionization. The dotted lines are those derived by Ahlborn (1966) for $r = 1$. The constant deviation between the curves for air derived from the theory presented in this note and that of Ahlborn's

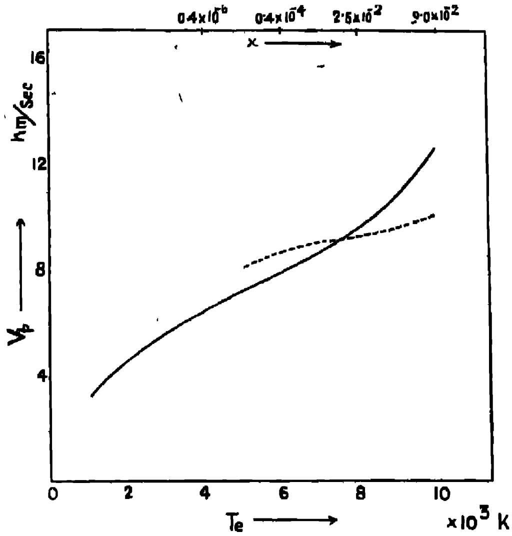

Fig. 2. Variation of sound velocity V_p with temperature of the plasma for hydrogen at 10^{-2} atmp. and $r = 1$ Dotted curve is that given by Klem's theory. Degree of ionization corresponding to the temperature of the plasma (hydrogen) is shown on the upper scale.

theory may be due to the variation in the values chosen by Ahlborn for density etc. at various temperatures and pressures given in Tables for air in the calculation of the total enthalpy. However the error between the two theories is about 10%. Figure 2 shows the variation of sound velocity with the degree of ionization for hydrogen at 10^{-2} atmosphere. The dotted curve is that given by the theory of Kelm (1973). Kelm's values differ by about 18%. However. it can be seen that the values derived from this theory deviate considerably from the other values for gas temperatures above 10^4 °K when the ionization is expected to be high. Hence it should be noted that the analysis presented here is applicable. only to plasmas when the ionization is low.

For hydrogen at 10^{-2} atm. pressure, for the same temperature the degree of ionization is high and hence the agreement between the curvses is not very-good as expected.

Ahlborn's theory for the velocity of sound in a plasma is based on the known values of gas enthalpy, density and pressure white Klem has derived the velocity of acoustic waves through a plasma from the knowledge of the variations in the coefficients of recombination and radiative collisional ionization. Variations in these coefficients with the plasma parameters such as n_e, n_0, T_e and T_0 is complex. The treatment presented in this note is different from the other two workers mentioned and is simple.

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References

Ahlborn B. 1966 *Phys. of Fluids* **9**, 1873.
Ingard U. 1966 *Phys. Rev.* **145**, 41.
Ingard U. & Schulz M. 1967 *Phys. Rev.* **158**, 106.
Kaw P. 1969 *Phys. Rev.* **188**, 506.
Kelm S. 1973 *Proc. 11th Intern. Conf. on Phenomena in Ionized Gases*, Prague, Czek. 317.
Schulz M. & Ingard U. 1969 *Phys. of Fluids*, **12**, 1237.
Sodha M. S. & Palumbo C. J. 1964 *Can. J. Phys.* **42**, 1635.
Von Engle A. 1965 *Ionized Gases*, Oxford University Press.